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A Novel Algorithm for Rate/Power Allocation in OFDM-based Cognitive Radio Systems with Statistical Interference Constraints

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Abstract—In this paper, we adopt a multiobjective optimization approach to jointly optimize the rate and power in OFDM-based cognitive radio (CR) systems. We propose a novel algorithm that jointly maximizes the OFDM-based CR system throughput and minimizes its transmit power, while guaranteeing a target bit error rate per subcarrier and a total transmit power threshold for the secondary user (SU), and restricting both co-channel and adjacent channel interferences to existing primary users (PUs) in a statistical manner. Since the interference constraints are met statistically, the SU transmitter does not require perfect channel-state-information (CSI) feedback from the PUs receivers. Closed-form expressions are derived for bit and power allocations per subcarrier. Simulation results illustrate the performance of the proposed algorithm and compare it to the case of perfect CSI. Further, the results show that the performance of the proposed algorithm approaches that of an exhaustive search for the discrete global optimal allocations with significantly reduced computational complexity.

Index Terms—Bit and power allocation, cognitive radio, dynamic spectrum sharing, statistical interference constraints.

I. INTRODUCTION

Cognitive radio (CR) can considerably enhance the spectrum utilization efficiency by dynamically sharing the spectrum between licensed/primary users (PUs) and unlicensed/secondary users (SUs) [1]. This is achieved by granting the SUs opportunistic access to the white spaces within the PUs spectrum, while controlling the interference to the PUs. Orthogonal frequency division multiplexing (OFDM) is recognized as an attractive modulation technique for CR due to its flexibility, adaptivity in allocating vacant radio resources, and spectrum shaping capabilities [1]. A common technique to improve the performance of the OFDM-based systems is to dynamically load different bits and/or powers per each subcarrier according to the wireless channel quality and the imposed PUs interference constraints [2]–[7].

The prior work in the literature focused on maximizing the OFDM SU capacity/throughput while limiting the interference introduced to PUs to a predefined threshold [2]–[7]. The authors in [2]–[4] consider perfect channel-state-information (CSI) between the SU transmitter and the PUs receivers, which is a challenging assumption for practical scenarios. In [5], [6], the authors assume only knowledge of the path loss for these links; however, such an assumption will cause the proposed algorithms to violate the interference constraints uncontrollably when applied in practice (i.e., since neither the instantaneous channel gains nor the channel statistics are known, there is no guarantee regarding the probability of viola-

tion of the interference constraints). The authors in [7] assume knowledge of the channel statistics (i.e., the fading distribution and its parameters), which is a reasonable assumption for certain wireless environments, e.g., in non-line-of-sight urban environments, a Rayleigh distribution is usually assumed for the magnitude of the fading channel coefficients. In this paper, we adopt the same channel assumption as in [7]; our main contributions when compared with the work in the literature are as follows: 1) a multiobjective optimization approach¹ is used for the dynamic spectrum sharing problem and 2) we guarantee a certain OFDM SU bit error rate (BER).

That being said, in this paper we propose a novel low-complexity algorithm for OFDM-based CR systems that jointly maximizes the OFDM SU throughput and minimizes its transmit power, subject to a target BER per subcarrier and total transmit power threshold for the SU, as well as statistical constraints on the co-channel interference (CCI) and adjacent channel interference (ACI) to the PUs. Closed-form expressions are derived for the bit and power allocations per subcarrier. Simulation results identify the performance degradation due to the incomplete channel information, by comparing the performance of the proposed algorithm with that of perfect CSI. Additionally, the results indicate that the performance of the proposed algorithm approaches that of an exhaustive search for the optimal allocations.

The remainder of the paper is organized as follows. Section II presents the system model and Section III introduces the proposed joint bit and power loading algorithm. Simulation results are presented in Section IV, while conclusions are drawn in Section V.

Throughout this paper we use bold-faced lower case letters for vectors, e.g., \mathbf{x} , and light-faced letters for scalar quantities, e.g., x . $[\cdot]^T$ denotes the transpose operation, ∇ represents the gradient operator, $\Pr(\cdot)$ denotes the probability, $\mathbb{E}[\cdot]$ is the statistical expectation operator, $[x, y]^-$ represents $\min(x, y)$, and \bar{X} is the cardinality of the set X .

II. SYSTEM MODEL

The available spectrum is assumed to be divided into \mathcal{M} subchannels that are licensed to \mathcal{M} PUs. A subchannel m , of

¹In a non-CR environment, jointly maximizing the throughput and minimizing the transmit power provides a significant performance improvement, in terms of the achieved throughput and transmit power, when compared to other work in the literature that separately maximizes the throughput (while constraining the transmit power) or minimizes the transmit power (while constraining the throughput), respectively [8].

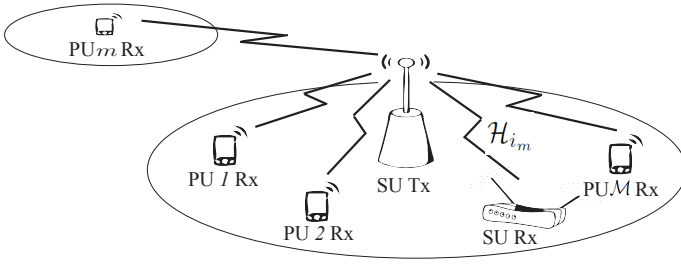


Fig. 1: Co-existence of an SU and M PUs in the spatial domain.

bandwidth B_m , has N_m subcarriers and i_m denotes subcarrier i in subchannel m , $i_m = 1, \dots, N_m$. A PU does not occupy its licensed spectrum all the time and/or at all its coverage locations; hence, a SU may access such voids as long as no harmful interference occurs to adjacent PUs due to ACI, or to other PUs operating in the same frequency band at distant locations due to CCI.

A typical CR system is shown in Fig. 1. The SU first obtains the surrounding PUs information, such as the PUs positions and spectral band occupancies². Then, it makes a decision on the possible transmission subchannels. We consider that the SU has all the required information on the existing M PUs, and it decides to use the vacant m th PU subchannel, $m \in \{1, \dots, M\}$.

Following the common practice in the literature, we assume that the instantaneous channel gains between the SU transmitter and receiver pairs are available through a delay- and error-free feedback channel [2]–[7]. Additionally, we assume that the SU transmitter has knowledge of the fading distribution type and its corresponding parameters of the channels $\mathcal{H}_{sp}^{(\ell)}$ and $\mathcal{H}_{sp}^{(m)}$ to the ℓ th and m th PUs receivers, respectively (given the fact that estimating the instantaneous channel gains between the SU transmitter and the PUs receivers is practically challenging).

The interference, \mathcal{I}_{i_m} , from all the PUs to subcarrier i_m of the SU is considered as in [3], [5]–[7], and depends on the SU receiver windowing function and power spectral density of the PUs. On the other hand, the ACI depends on the power allocated to each SU subcarrier and the spectral distance between the SU subcarriers and the PUs. The ACI from the SU to the ℓ th PU receiver is formulated as [3], [5]–[7]

$$|\mathcal{H}_{sp}^{(\ell)}|^2 \sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \varpi_{i_m}^{(\ell)} \leq \mathcal{P}_{ACI}^{(\ell)}, \quad (1)$$

where $\varpi_{i_m}^{(\ell)} = T_{s,m} 10^{-0.1L(d_\ell)} \int_{f_{i_m,\ell}-\frac{B_\ell}{2}}^{f_{i_m,\ell}+\frac{B_\ell}{2}} \text{sinc}^2(T_{s,m}f) df$, $T_{s,m}$ is the duration of the OFDM symbol of the SU, $L(d_\ell)$ is the path loss in dB at distance d_ℓ , d_ℓ is the distance between the SU and the ℓ th PU receiver, $f_{i_m,\ell}$ is the spectral distance between the SU subcarrier i_m and the ℓ th PU receiver frequency band, B_ℓ is the bandwidth of the ℓ th PU receiver, \mathcal{P}_{i_m} is the transmit power per subcarrier i_m , $\mathcal{P}_{ACI}^{(\ell)}$ is the interference threshold at the ℓ th PU receiver, and $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$.

The CCI at the location of the distant m th PU receiver is required to be limited as

$$|\mathcal{H}_{sp}^{(m)}|^2 10^{-0.1L(d_m)} \sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \leq \mathcal{P}_{CCI}^{(m)}, \quad (2)$$

where $\mathcal{P}_{CCI}^{(m)}$ is the interference threshold at the m th PU. To reflect the SU transmitter's power amplifier limitations or/and to satisfy regulatory maximum power limits, the total SU transmit power is limited to a certain threshold \mathcal{P}_{th} as

$$\sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \leq \mathcal{P}_{th}. \quad (3)$$

III. PROPOSED ALGORITHM

A. Optimization Problem Formulation

We propose a novel close-to-optimal algorithm that jointly maximizes the OFDM SU throughput and minimizes its transmit power, while satisfying a target BER per subcarrier³ and a total transmit power threshold for the SU, and limiting the introduced CCI and ACI to the m th and ℓ th PUs receivers below the thresholds $\mathcal{P}_{CCI}^{(m)}$ and $\mathcal{P}_{CCI}^{(\ell)}$ with at least a probability of $\Psi_{CCI}^{(m)}$ and $\Psi_{ACI}^{(\ell)}$, respectively. The optimization problem is formulated as

$$\begin{aligned} & \underset{\mathcal{P}_{i_m}}{\text{Minimize}} \sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \text{ and } \underset{b_{i_m}}{\text{Maximize}} \sum_{i_m=1}^{N_m} b_{i_m}, \\ & \text{subject to} \quad \text{BER}_{i_m} \leq \text{BER}_{th,i_m}, \end{aligned} \quad (4a)$$

$$\sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \leq \mathcal{P}_{th}, \quad (4b)$$

$$\Pr \left(|\mathcal{H}_{sp}^{(m)}|^2 10^{-0.1L(d_m)} \sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \leq \mathcal{P}_{CCI}^{(m)} \right) \geq \Psi_{CCI}^{(m)}, \quad (4c)$$

$$\Pr \left(|\mathcal{H}_{sp}^{(\ell)}|^2 \sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \varpi_{i_m}^{(\ell)} \leq \mathcal{P}_{ACI}^{(\ell)} \right) \geq \Psi_{ACI}^{(\ell)}, \quad (4d)$$

where $i_m = 1, \dots, N_m$, $\ell = 1, \dots, M$, b_{i_m} is the number of bits per subcarrier i_m , and BER_{i_m} and BER_{th,i_m} are the BER per subcarrier i_m and the threshold value of the BER per subcarrier i_m , respectively.

A non-line-of-sight propagation environment is assumed; therefore, the channel gains $\mathcal{H}_{sp}^{(m)}$ and $\mathcal{H}_{sp}^{(\ell)}$ can be modeled as zero-mean complex Gaussian random variables, and, hence, $|\mathcal{H}_{sp}^{(m)}|^2$ and $|\mathcal{H}_{sp}^{(\ell)}|^2$ follow an exponential distribution [7]. Accordingly, the statistical CCI interference constraint in (4c) can be evaluated as

$$1 - \exp \left(-\frac{\nu}{10^{-0.1L(d_m)} \sum_{i_m=1}^{N_m} \mathcal{P}_{i_m}} \mathcal{P}_{CCI}^{(m)} \right) \geq \Psi_{CCI}^{(m)}, \quad (5)$$

where $\frac{1}{\nu}$ is the mean of the exponential distribution. Eqn. (5) can be further written as

$$\sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \leq \frac{\nu 10^{0.1L(d_m)} \mathcal{P}_{CCI}^{(m)}}{-\ln(1 - \Psi_{CCI}^{(m)})}, \quad (6)$$

and the constraints in (4b) and (4c) can be combined as

$$\sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \leq \left[\mathcal{P}_{th}, \frac{\nu 10^{0.1L(d_m)} \mathcal{P}_{CCI}^{(m)}}{-\ln(1 - \Psi_{CCI}^{(m)})} \right]^-. \quad (7)$$

³The constraint on the BER per subcarrier is a suitable formulation that results in similar BER characteristics when compared with an average BER constraint, especially at high signal-to-noise ratios (SNR) [9]. Furthermore, it facilitates derivation of closed-form expressions for the optimal bit and power solutions, which reduces the algorithmic complexity.

²This is done by visiting a database administrated by a government or third party, or by optionally sensing and determining the PUs radio frequency and positions, respectively [5].

Similarly, the statistical ACI constraint in (4d) is rewritten as

$$\sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \varpi_{i_m}^{(\ell)} \leq \frac{\nu}{-\ln(1 - \Psi_{\text{ACI}}^{(\ell)})} \mathcal{P}_{\text{CCI}}^{(\ell)}. \quad (8)$$

An approximate expression for the BER per subcarrier i_m in the case of M -ary QAM [10], while taking the interference from the PUs into account, is given by

$$\text{BER}_{i_m} \approx 0.2 \exp \left(-1.6 \frac{\mathcal{P}_{i_m}}{(2^{b_{i_m}} - 1)} \frac{|\mathcal{H}_{i_m}|^2}{(\sigma_n^2 + \mathcal{J}_{i_m})} \right), \quad (9)$$

where \mathcal{H}_{i_m} is the channel gain of subcarrier i_m between the SU transmitter and receiver pair and σ_n^2 is the variance of the additive white Gaussian noise (AWGN).

The multiobjective optimization problem in (4) can be rewritten as a linear combination of the multiple objectives as

$$\begin{aligned} \text{Minimize}_{\mathcal{P}_{i_m}, b_{i_m}} \quad & \mathcal{F}(\mathbf{p}_m, \mathbf{b}_m) = \alpha \sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} - (1 - \alpha) \sum_{i_m=1}^{N_m} b_{i_m}, \\ \text{subject to} \quad & g_\rho(\mathbf{p}_m, \mathbf{b}_m) \leq 0, \end{aligned} \quad (10)$$

where α ($0 < \alpha < 1$) is a constant which indicates the relative importance of one objective function relative to the other, being selected according to the CR requirements/applications, i.e., minimum power versus maximum throughput, $\rho = 1, \dots, N_m + 2$ is the constraint index, $\mathbf{p}_m = [\mathcal{P}_{1_m}, \dots, \mathcal{P}_{N_m}]^T$ and $\mathbf{b}_m = [b_{1_m}, \dots, b_{N_m}]^T$ are the N_m -dimensional power and bit distribution vectors, respectively, and

$$g_\rho(\mathbf{p}_m, \mathbf{b}_m) = \begin{cases} 0.2 \sum_{i_m=1}^{N_m} \exp \left(\frac{-1.6 \mathcal{C}_{i_m} \mathcal{P}_{i_m}}{2^{b_{i_m}} - 1} \right) - \text{BER}_{th, i_m} \leq 0, & \rho = i_m = 1, \dots, N_m, \\ \sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} - \left[\mathcal{P}_{th}, \frac{\nu 10^{0.1L(d_m)}}{-\ln(1 - \Psi_{\text{CCI}}^{(m)})} \mathcal{P}_{\text{CCI}}^{(m)} \right]^- \leq 0, & \rho = N_m + 1, \\ \sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \varpi_{i_m}^{(\ell)} - \frac{\nu}{-\ln(1 - \Psi_{\text{ACI}}^{(\ell)})} \mathcal{P}_{\text{CCI}}^{(\ell)} \leq 0, & \rho = N_m + 2, \end{cases} \quad (11)$$

where $\ell = 1, \dots, \mathcal{M}$ and $\mathcal{C}_{i_m} = \frac{|\mathcal{H}_{i_m}|^2}{\sigma_n^2 + \mathcal{J}_{i_m}}$ is the channel-to-noise-plus-interference ratio for subcarrier i_m .

B. Optimization Problem Analysis and Solution

The optimization problem in (10) can be solved by the method of Lagrange multipliers. Accordingly, the inequality constraints are transformed to equality constraints by adding non-negative slack variables, \mathcal{Y}_ρ^2 , $\rho = 1, \dots, N_m + 2$ [11]. Hence, the constraints are given as

$$\mathcal{G}_\rho(\mathbf{p}_m, \mathbf{b}_m, \mathbf{y}) = g_\rho(\mathbf{p}_m, \mathbf{b}_m) + \mathcal{Y}_\rho^2 = 0, \quad (12)$$

where $\mathbf{y} = [\mathcal{Y}_1^2, \dots, \mathcal{Y}_{N_m+2}^2]^T$ is the vector of slack variables, and the Lagrangian function \mathcal{L} is expressed as

$$\begin{aligned} \mathcal{L}(\mathbf{p}_m, \mathbf{b}_m, \mathbf{y}, \boldsymbol{\lambda}) &= \mathcal{F}(\mathbf{p}_m, \mathbf{b}_m) + \sum_{\rho=1}^{N_m+2} \lambda_\rho \mathcal{G}_\rho(\mathbf{p}_m, \mathbf{b}_m, \mathbf{y}) \\ &= \alpha \sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} - (1 - \alpha) \sum_{i_m=1}^{N_m} b_{i_m} \\ &+ \sum_{i_m=1}^{N_m} \lambda_{i_m} \left[0.2 \exp \left(\frac{-1.6 \mathcal{C}_{i_m} \mathcal{P}_{i_m}}{2^{b_{i_m}} - 1} \right) - \text{BER}_{th, i_m} + \mathcal{Y}_{i_m}^2 \right] \\ &+ \lambda_{N_m+1} \left[\sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} - \left[\mathcal{P}_{th}, \frac{\nu 10^{0.1L(d_m)}}{-\ln(1 - \Psi_{\text{CCI}}^{(m)})} \mathcal{P}_{\text{CCI}}^{(m)} \right]^- + \mathcal{Y}_{N_m+1}^2 \right] \\ &+ \sum_{\ell=1}^{\mathcal{M}} \lambda_{N_m+2}^{(\ell)} \left[\sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \varpi_{i_m}^{(\ell)} - \frac{\nu}{-\ln(1 - \Psi_{\text{ACI}}^{(\ell)})} \mathcal{P}_{\text{CCI}}^{(\ell)} + \mathcal{Y}_{N_m+2}^{2,(\ell)} \right], \end{aligned} \quad (13)$$

where $\boldsymbol{\lambda} = [\lambda_1, \dots, \lambda_{N_m+2}^{(\ell)}]^T$ is the vector of Lagrange multipliers associated with the $N_m + 2$ constraints in (11). A stationary point is found when $\nabla \mathcal{L}(\mathbf{p}_m, \mathbf{b}_m, \mathbf{y}, \boldsymbol{\lambda}) = 0$, which yields

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \mathcal{P}_{i_m}} &= \alpha - \lambda_{i_m} \frac{(0.2)(1.6) \mathcal{C}_{i_m}}{2^{b_{i_m}} - 1} \exp \left(\frac{-1.6 \mathcal{C}_{i_m} \mathcal{P}_{i_m}}{2^{b_{i_m}} - 1} \right) \\ &+ \lambda_{N_m+1} + \sum_{\ell=1}^{\mathcal{M}} \varpi_{i_m}^{(\ell)} \lambda_{N_m+2}^{(\ell)} = 0, \end{aligned} \quad (14a)$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial b_{i_m}} &= -(1 - \alpha) + \lambda_{i_m} \frac{(0.2)(1.6)(\ln(2)) \mathcal{C}_{i_m} \mathcal{P}_{i_m} 2^{b_{i_m}}}{(2^{b_{i_m}} - 1)^2} \\ &\exp \left(\frac{-1.6 \mathcal{C}_{i_m} \mathcal{P}_{i_m}}{2^{b_{i_m}} - 1} \right) = 0, \end{aligned} \quad (14b)$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \lambda_{i_m}} &= 0.2 \exp \left(\frac{-1.6 \mathcal{C}_{i_m} \mathcal{P}_{i_m}}{2^{b_{i_m}} - 1} \right) - \text{BER}_{th, i_m} + \mathcal{Y}_{i_m}^2 \\ &= 0, \end{aligned} \quad (14c)$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \lambda_{N_m+1}} &= \sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} - \left[\mathcal{P}_{th}, \frac{\nu 10^{0.1L(d_m)}}{-\ln(1 - \Psi_{\text{CCI}}^{(m)})} \mathcal{P}_{\text{CCI}}^{(m)} \right]^- \\ &+ \mathcal{Y}_{N_m+1}^2 = 0, \end{aligned} \quad (14d)$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \lambda_{N_m+2}^{(\ell)}} &= \sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \varpi_{i_m}^{(\ell)} - \frac{\nu}{-\ln(1 - \Psi_{\text{ACI}}^{(\ell)})} \mathcal{P}_{\text{CCI}}^{(\ell)} \\ &+ \mathcal{Y}_{N_m+2}^{2,(\ell)} = 0, \end{aligned} \quad (14e)$$

$$\frac{\partial \mathcal{L}}{\partial \mathcal{Y}_{i_m}^2} = 2\lambda_{i_m} \mathcal{Y}_{i_m} = 0, \quad (14f)$$

$$\frac{\partial \mathcal{L}}{\partial \mathcal{Y}_{N_m+1}^2} = 2\lambda_{N_m+1} \mathcal{Y}_{N_m+1} = 0, \quad (14g)$$

$$\frac{\partial \mathcal{L}}{\partial \mathcal{Y}_{N_m+2}^{2,(\ell)}} = 2\lambda_{N_m+2}^{(\ell)} \mathcal{Y}_{N_m+2}^{2,(\ell)} = 0. \quad (14h)$$

It can be seen that (14a)-(14h) represent $4N_m + 2\mathcal{M} + 2$ equations in the $4N_m + 2\mathcal{M} + 2$ unknown components of the vectors $\mathbf{p}_m, \mathbf{b}_m, \mathbf{y}$, and $\boldsymbol{\lambda}$. By solving (14), one obtains the solution $\mathbf{p}_m^*, \mathbf{b}_m^*$. Equation (14f) implies that either $\lambda_{i_m} = 0$ or $\mathcal{Y}_{i_m} = 0$, (14g) implies that either $\lambda_{N_m+1} = 0$ or $\mathcal{Y}_{N_m+1} = 0$, and (14h) implies that either $\lambda_{N_m+2}^{(\ell)} = 0$ or $\mathcal{Y}_{N_m+2}^{2,(\ell)} = 0$. Hence, eight possible cases exist and we are going to investigate each case independently.

— *Cases 1, 2, 3, and 4:* In (14), setting $\lambda_{i_m} = 0$ and $\lambda_{N_m+1} = 0$ (case 1)/ $\mathcal{Y}_{N_m+1} = 0$ (case 2), or $\lambda_{N_m+2}^{(\ell)} = 0$ (case 3)/ $\mathcal{Y}_{N_m+2}^{2,(\ell)} = 0$ (case 4) results in an underdetermined system, and, hence, no unique solution can be reached.

— *Case 5:* Setting $\mathcal{Y}_{i_m} = 0$, $\lambda_{N_m+1} = 0$ (i.e., inactive CCI/total transmit power constraint), and $\lambda_{N_m+2}^{(\ell)} = 0$ (i.e., inactive ACI constraint), we can relate \mathcal{P}_{i_m} and b_{i_m} from (14a) and (14b) as

$$\mathcal{P}_{i_m} = \frac{1 - \alpha}{\alpha \ln(2)} (1 - 2^{-b_{i_m}}), \quad (15)$$

with $\mathcal{P}_{i_m} \geq 0$ if and only if $b_{i_m} \geq 0$. By substituting (15) into (14c), one obtains the solution

$$b_{i_m}^* = \log_2 \left[-\frac{1 - \alpha}{\alpha \ln(2)} \frac{1.6 \mathcal{C}_{i_m}}{\ln(5 \text{BER}_{th, i_m})} \right]. \quad (16)$$

Consequently, from (15) one gets

$$\mathcal{P}_{i_m}^* = \frac{1 - \alpha}{\alpha \ln(2)} + \frac{\ln(5 \text{BER}_{th, i_m})}{1.6 \mathcal{C}_{i_m}}. \quad (17)$$

Since we consider M -ary QAM, b_{i_m} should be greater than 2. From (16), to have $b_{i_m} \geq 2$, \mathcal{C}_{i_m} , must satisfy the condition

$$\mathcal{C}_{i_m} \geq \mathcal{C}_{th,i_m} = -\frac{4}{1.6} \frac{\alpha \ln(2)}{1-\alpha} \ln(5 \text{BER}_{th,i_m}), i_m = 1, \dots, N_m. \quad (18)$$

— *Case 6:* Setting $\mathcal{Y}_{i_m} = 0$, $\mathcal{Y}_{N_m+1} = 0$ (i.e., active CCI/total transmit power constraint), and $\lambda_{N_m+2}^{(\ell)} = 0$ (i.e., inactive ACI constraint), similar to *case 5*, we obtain

$$\mathcal{P}_{i_m} = \frac{1-\alpha}{\ln(2)(\alpha + \lambda_{N_m+1})} (1 - 2^{-b_{i_m}}), \quad (19)$$

$$b_{i_m}^* = \log_2 \left[-\frac{1-\alpha}{\ln(2)(\alpha + \lambda_{N_m+1})} \frac{1.6 \mathcal{C}_{i_m}}{\ln(5 \text{BER}_{th,i_m})} \right]. \quad (20)$$

$$\mathcal{P}_{i_m}^* = \frac{1-\alpha}{\ln(2)(\alpha + \lambda_{N_m+1})} + \frac{\ln(5 \text{BER}_{th,i_m})}{1.6 \mathcal{C}_{i_m}}, \quad (21)$$

where λ_{N_m+1} is calculated to satisfy the active CCI/total transmit power constraint in (14d). Hence, the value of λ_{N_m+1} is found to be

$$\lambda_{N_m+1} = \frac{\bar{N}_m^a \frac{1-\alpha}{\ln 2}}{\left[\mathcal{P}_{th}, \frac{\nu 10^{0.1L(d_m)}}{-\ln(1-\Psi_{\text{CCI}}^{(m)})} \mathcal{P}_{\text{CCI}}^{(m)} \right]^- - \sum_{i_m \in \bar{N}_m^a} \frac{\ln(5 \text{BER}_{th,i_m})}{1.6 \mathcal{C}_{i_m}}} - \alpha, \quad (22)$$

where \bar{N}_m^a is the cardinality of the set of active subcarriers \bar{N}_m^a .

— *Case 7:* Setting $\mathcal{Y}_{i_m} = 0$, $\lambda_{N_m+1} = 0$ (i.e., inactive CCI/total transmit power constraint), and $\mathcal{Y}_{N_m+2}^{(\ell)} = 0$ (i.e., active ACI constraint), similar to *cases 5* and *6*, we obtain

$$\mathcal{P}_{i_m} = \frac{1-\alpha}{\ln(2)(\alpha + \sum_{\ell=1}^M \varpi_{i_m}^{(\ell)} \lambda_{N_m+2}^{(\ell)})} (1 - 2^{-b_{i_m}}), \quad (23)$$

$$b_{i_m}^* = \log_2 \left[-\frac{1-\alpha}{\ln(2)(\alpha + \sum_{\ell=1}^M \varpi_{i_m}^{(\ell)} \lambda_{N_m+2}^{(\ell)})} \frac{1.6 \mathcal{C}_{i_m}}{\ln(5 \text{BER}_{th,i_m})} \right]. \quad (24)$$

$$\mathcal{P}_{i_m}^* = \frac{1-\alpha}{\ln(2)(\alpha + \sum_{\ell=1}^M \varpi_{i_m}^{(\ell)} \lambda_{N_m+2}^{(\ell)})} + \frac{\ln(5 \text{BER}_{th,i_m})}{1.6 \mathcal{C}_{i_m}}. \quad (25)$$

where $\lambda_{N_m+2}^{(\ell)}$ is calculated numerically using the Newton's method to satisfy the active ACI constraint in (14e).

— *Case 8:* Setting $\mathcal{Y}_{i_m} = 0$, $\mathcal{Y}_{N_m+1} = 0$ (i.e., active CCI/total transmit power constraint), and $\mathcal{Y}_{N_m+2}^{(\ell)} = 0$ (i.e., active ACI constraint), similar to the previous *cases*, we obtain

$$\mathcal{P}_{i_m} = \frac{1-\alpha}{\ln(2)(\alpha + \lambda_{N_m+1} + \sum_{\ell=1}^M \varpi_{i_m}^{(\ell)} \lambda_{N_m+2}^{(\ell)})} (1 - 2^{-b_{i_m}}), \quad (26)$$

$$b_{i_m}^* = \log_2 \left[-\frac{1-\alpha}{\ln(2)(\alpha + \lambda_{N_m+1} + \sum_{\ell=1}^M \varpi_{i_m}^{(\ell)} \lambda_{N_m+2}^{(\ell)})} \frac{1.6 \mathcal{C}_{i_m}}{\ln(5 \text{BER}_{th,i_m})} \right], \quad (27)$$

$$\mathcal{P}_{i_m}^* = \frac{1-\alpha}{\ln(2)(\alpha + \lambda_{N_m+1} + \sum_{\ell=1}^M \varpi_{i_m}^{(\ell)} \lambda_{N_m+2}^{(\ell)})} + \frac{\ln(5 \text{BER}_{th,i_m})}{1.6 \mathcal{C}_{i_m}}, \quad (28)$$

where λ_{N_m+1} and $\lambda_{N_m+2}^{(\ell)}$ are calculated numerically to satisfy the active CCI/total transmit power and ACI constraints in (14d) and (14e), respectively.

The obtained solution $(\mathbf{p}_m^*, \mathbf{b}_m^*)$ represents a minimum of $\mathcal{F}(\mathbf{p}_m, \mathbf{b}_m)$ as the Karush-Kuhn-Tucker (KKT) conditions [11] are satisfied; the proof is not included due to space limitations. Please note that the optimization problem in (10)

is not convex, and the obtained solution is not guaranteed to be a global optimum. In the next section, we compare the local optimum results to the global optimum results achieved through an exhaustive search to 1) characterize the gap to the global optimum solution and 2) characterize the gap to the equivalent discrete optimization problem (i.e., with integer constraints on b_{i_m}).

C. Proposed Joint Bit and Power Loading Algorithm

The proposed algorithm can be formally stated as follows

Proposed Algorithm

- 1: **INPUT** σ_n^2 , \mathcal{H}_{i_m} , BER_{th,i_m} , α , \mathcal{P}_{th} , $\mathcal{P}_{\text{CCI}}^{(m)}$, $\mathcal{P}_{\text{CCI}}^{(\ell)}$, ν , $\Psi_{\text{CCI}}^{(m)}$, $\Psi_{\text{ACI}}^{(\ell)}$, and PUs information.
- 2: **for** $i_m = 1, \dots, N_m$ **do**
- 3: **if** $\mathcal{C}_{i_m} \geq \mathcal{C}_{th,i_m} = -\frac{4}{1.6} \frac{\alpha \ln(2)}{1-\alpha} \ln(5 \text{BER}_{th,i_m})$ **then**
- 4: $b_{i_m}^*$ and $\mathcal{P}_{i_m}^*$ are given by (16) and (17), respectively.
- 5: **else**
- 6: Null the corresponding subcarrier i_m .
- 7: **end if**
- 8: **end for**
- 9: **if** $\sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \geq \left[\mathcal{P}_{th}, \frac{\nu 10^{0.1L(d_m)}}{-\ln(1-\Psi_{\text{CCI}}^{(m)})} \mathcal{P}_{\text{CCI}}^{(m)} \right]^-$ and $\sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \varpi_{i_m}^{(\ell)} \leq \frac{\nu}{-\ln(1-\Psi_{\text{ACI}}^{(\ell)})} \mathcal{P}_{\text{CCI}}^{(\ell)}$ **then**
- 10: $b_{i_m}^*$ and $\mathcal{P}_{i_m}^*$ are given by (20) and (21), respectively.
- 11: λ_{N_m+1} is given by (22) and $\lambda_{N_m+2}^{(\ell)} = 0$.
- 12: **else if** $\sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \leq \left[\mathcal{P}_{th}, \frac{\nu 10^{0.1L(d_m)}}{-\ln(1-\Psi_{\text{CCI}}^{(m)})} \mathcal{P}_{\text{CCI}}^{(m)} \right]^-$ and $\sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \varpi_{i_m}^{(\ell)} \geq \frac{\nu}{-\ln(1-\Psi_{\text{ACI}}^{(\ell)})} \mathcal{P}_{\text{CCI}}^{(\ell)}$ **then**
- 13: $b_{i_m}^*$ and $\mathcal{P}_{i_m}^*$ are given by (24) and (25), respectively.
- 14: $\lambda_{N_m+1} = 0$ and $\lambda_{N_m+2}^{(\ell)}$ are calculated to satisfy $\sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \varpi_{i_m}^{(\ell)} = \frac{\nu}{-\ln(1-\Psi_{\text{ACI}}^{(\ell)})} \mathcal{P}_{\text{CCI}}^{(\ell)}$
- 15: **else if** $\sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \geq \left[\mathcal{P}_{th}, \frac{\nu 10^{0.1L(d_m)}}{-\ln(1-\Psi_{\text{CCI}}^{(m)})} \mathcal{P}_{\text{CCI}}^{(m)} \right]^-$ and $\sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \varpi_{i_m}^{(\ell)} \geq \frac{\nu}{-\ln(1-\Psi_{\text{ACI}}^{(\ell)})} \mathcal{P}_{\text{CCI}}^{(\ell)}$ **then**
- 16: $b_{i_m}^*$ and $\mathcal{P}_{i_m}^*$ are given by (27) and (28), respectively.
- 17: λ_{N_m+1} and $\lambda_{N_m+2}^{(\ell)}$ are calculated to satisfy $\sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} = \left[\mathcal{P}_{th}, \frac{\nu 10^{0.1L(d_m)}}{-\ln(1-\Psi_{\text{CCI}}^{(m)})} \mathcal{P}_{\text{CCI}}^{(m)} \right]^-$ and $\sum_{i_m=1}^{N_m} \mathcal{P}_{i_m} \varpi_{i_m}^{(\ell)} = \frac{\nu}{-\ln(1-\Psi_{\text{ACI}}^{(\ell)})} \mathcal{P}_{\text{CCI}}^{(\ell)}$, respectively.
- 18: **end if**
- 19: $b_{i_m,final}^* \leftarrow \text{Round } b_{i_m}^*$ to the nearest integer.
- 20: $\mathcal{P}_{i_m,final}^* \leftarrow \text{Recalculate } \mathcal{P}_{i_m}^*$ according to (9).
- 21: If the conditions on the CCI/total transmit power and the ACI are violated due to rounding, decrement the number of bits on the subcarrier that has the largest $\Delta \mathcal{P}_{i_m}(b_{i_m}) = \mathcal{P}_{i_m}(b_{i_m}) - \mathcal{P}_{i_m}(b_{i_m} - 1)$ until satisfied.
- 22: **OUTPUT** $b_{i_m,final}^*$ and $\mathcal{P}_{i_m,final}^*$, $i_m = 1, \dots, N_m$.

IV. NUMERICAL RESULTS

In this section, we present illustrative numerical results for the proposed allocation algorithm. Without loss of generality, we assume that the OFDM SU coexists with one adjacent PU and one co-channel PU. The OFDM SU transmission parameters are as follows: number of subcarriers $N_m = 128$, symbol duration $T_{s,m} = 102.4 \mu\text{sec}$, and subcarrier spacing $\Delta f_m = 9.7656 \text{ kHz}$. The path loss parameters are as follows: exponent = 4, wavelength = 0.33 meters, distance to the ℓ th PU receiver $d_\ell = 1 \text{ km}$, distance to the m th PU receiver $d_m = 5 \text{ km}$, and reference distance $d_0 = 500 \text{ m}$. BER_{th,i_m} is assumed to be the same for all subcarriers and set to

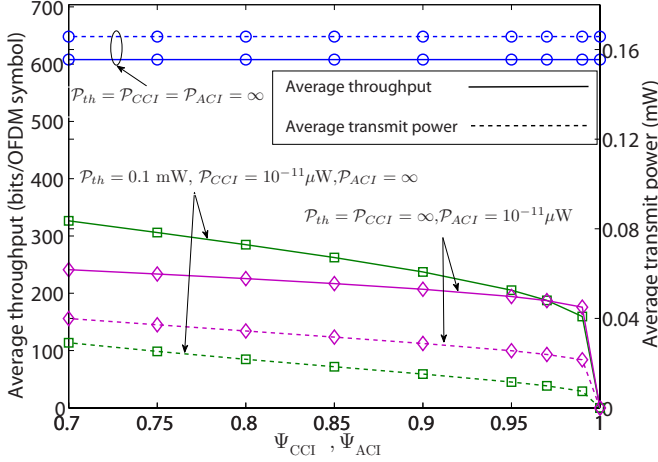


Fig. 2: Effect of Ψ_{CCI} and Ψ_{ACI} on the SU performance for different values of P_{th} , P_{CCI} , and P_{ACI} .

10^{-4} . σ_n^2 is assumed to be $10^{-3} \mu\text{W}$ and the PUs signals are assumed to be elliptically-filtered white random processes [3], [5]–[7]. Representative results are presented in this section, which were obtained through Monte Carlo trials for 10^4 channel realizations. Unless otherwise mentioned, $\alpha = 0.5$ and $\Psi_{CCI} = \Psi_{ACI} = 0.9$.

In Fig. 2, the average throughput and transmit power are plotted as a function of the probabilities Ψ_{CCI} and Ψ_{ACI} , for different values of P_{th} , P_{CCI} , and P_{ACI} . As expected, for $P_{th} = P_{CCI} = \infty$ and $P_{ACI} = \infty$, increasing the value of Ψ_{CCI} and Ψ_{ACI} has no effect on the achieved average throughput and transmit power, as the CCI and ACI constraints are inactive. For other values of P_{th} , P_{CCI} , and P_{ACI} , increasing the value of the probabilities Ψ_{CCI} and Ψ_{ACI} , slightly decreases the achieved average throughput and transmit power in order to meet such tight statistical constraints (i.e., meeting the CCI and ACI constraints with higher probabilities). The achieved average throughput and transmit power drop to zero for $\Psi_{CCI} = \Psi_{ACI} = 1$ as the proposed algorithm cannot meet such stringent requirements of satisfying the active CCI and the ACI constraints all the time, without knowledge of the instantaneous channel gains.

Fig. 3 shows the average throughput and transmit power as a function of the weighting factor α , for different values of P_{th} , P_{CCI} , and P_{ACI} . For $P_{th} = P_{CCI} = \infty$ and $P_{ACI} = \infty$, one can notice that an increase of the weighting factor α yields a decrease of both the average throughput and transmit power. This can be explained as follows: by increasing α , more weight is given to the transmit power minimization (the minimum transmit power is further reduced), whereas less weight is given to the throughput maximization (the maximum throughput is reduced), according to the problem formulation. Similar behaviour is noticed for $P_{th} = P_{CCI} = \infty$ and $P_{ACI} = 10^{-8} \mu\text{W}$ with reduced values of the average throughput and transmit power for lower values of α due to the active ACI constraint. For $P_{th} = 0.1 \text{ mW}$ and $P_{CCI} = 10^{-8} \mu\text{W}$ and $P_{ACI} = \infty$, the average throughput and transmit power are similar to their respective values if the total transmit power is less than $[P_{th}, \frac{\nu 10^{0.1L(d_m)}}{-\ln(1-\Psi_{CCI})} P_{CCI}]^- =$

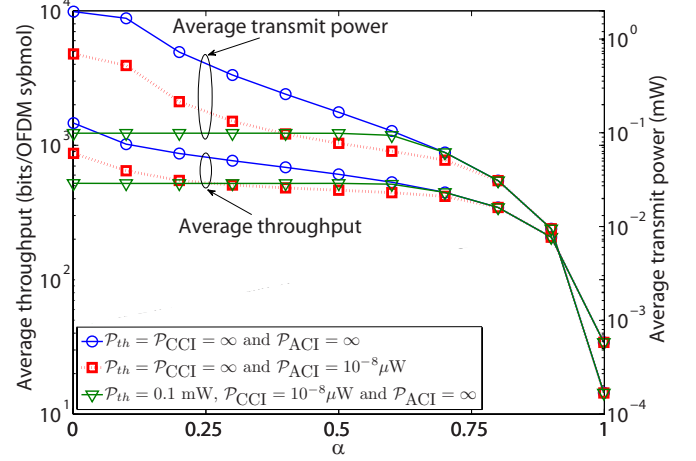


Fig. 3: Effect of α on the SU performance for different values of P_{th} , P_{CCI} , and P_{ACI} .

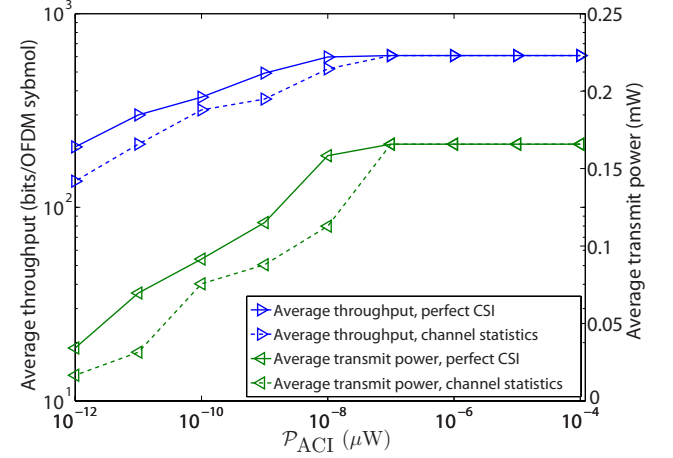


Fig. 4: Effect of P_{ACI} on the SU performance for $P_{th} = P_{CCI} = \infty$, with perfect CSI and channel statistics, respectively.

$[0.1 \text{ mW}, 15.4307 \text{ mW}]^- = 0.1 \text{ mW}$, while they saturate if the total transmit power exceeds 0.1 mW . Fig. 3 illustrates the benefit of introducing such a weighting factor in our problem formulation to tune the average throughput and transmit power levels as needed by the CR system.

Fig. 4 depicts the average throughput and transmit power as a function of the ACI threshold P_{ACI} , for $P_{th} = P_{CCI} = \infty$ and with knowledge of the perfect CSI and channel statistics, respectively. As can be seen for both cases of channel knowledge, the average throughput and transmit power increase as P_{ACI} increases, and saturate for higher values of P_{ACI} . This behaviour can be explained, as for lower values of P_{ACI} the ACI constraint is active and it affects the total transmit power. Increasing P_{ACI} results in a corresponding increase in both the average throughput and total transmit power. For higher values of P_{ACI} , the ACI constraint is inactive and the achieved throughput and transmit power saturate. As expected, the same performance is achieved for both perfect CSI and channel statistics knowledge for higher values of P_{ACI} ; this is because the ACI constraint is inactive, i.e., $P_{ACI} = \infty$ (please note that the CCI is inactive), and it will not be violated regardless of the channel knowledge. On the other hand, for lower values of P_{ACI} and with only knowledge of the channel statistics,

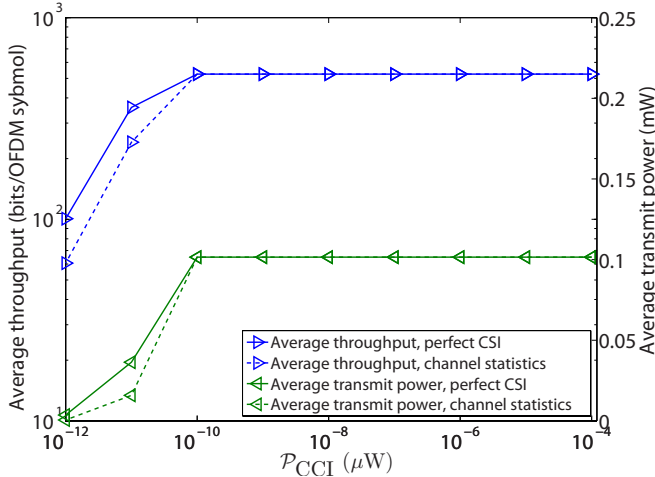


Fig. 5: Effect of \mathcal{P}_{CCI} on the SU performance for $\mathcal{P}_{th} = 0.1$ mW and $\mathcal{P}_{ACI} = \infty$, with perfect CSI and channel statistics, respectively.

the achieved average throughput and transmit power degrade when compared to the case of perfect CSI.

In Fig. 5, we plot the average throughput and transmit power as a function of the CCI threshold \mathcal{P}_{CCI} , for $\mathcal{P}_{th} = 0.1$ mW and $\mathcal{P}_{ACI} = \infty$, and with knowledge of the perfect CSI and channel statistics, respectively. As can be seen for both cases of channel knowledge, the average throughput and transmit power increase as \mathcal{P}_{CCI} increases, and saturate for higher values of \mathcal{P}_{CCI} . This can be explained, as for lower values of \mathcal{P}_{CCI} , $[0.1 \text{ mW}, \frac{\nu 10^{0.1L(d_m)}}{-\ln(1-\Psi_{CCI})} \mathcal{P}_{CCI}]^- = \frac{\nu 10^{0.1L(d_m)}}{-\ln(1-\Psi_{CCI})} \mathcal{P}_{CCI}$. Hence, the CCI constraint is active and affects the total transmit power. Increasing \mathcal{P}_{CCI} results in a corresponding increase in both the average throughput and transmit power. For higher values of \mathcal{P}_{CCI} , $[0.1 \text{ mW}, \frac{\nu 10^{0.1L(d_m)}}{-\ln(1-\Psi_{CCI})} \mathcal{P}_{CCI}]^- = 0.1 \text{ mW}$ and the transmit power is limited by the value of $\mathcal{P}_{th} = 0.1$ mW, while the achieved throughput saturates accordingly. Similar to the discussion in Fig. 4, the performance degrades for the case when only the channel statistics are known if the CCI constraint is active, i.e., at lower values of \mathcal{P}_{CCI} . On the other hand, the same performance is achieved for both cases of the channel knowledge for higher values of \mathcal{P}_{CCI} (please note that the ACI constraint is inactive).

Fig. 6 compares the objective function achieved with the proposed algorithm and an exhaustive search that finds the discretized global optimal allocation for the problem in (10) for $\mathcal{P}_{th} = 5$ μ W, $\mathcal{P}_{CCI} = \mathcal{P}_{ACI} = 10^{-10}$ μ W. Results are presented for a small number of subcarriers $N_m = 4, 6$, and 8, such that the exhaustive search is feasible. The exhaustive search tests all possible combinations of the bit and power allocations (the power per subcarrier is calculated from the discrete value of the bit allocation and $\text{BER}_{th,i}$) and selects the pair with the least objective function value. As one can notice, the proposed algorithm approaches the optimal results of the exhaustive search. The computational complexity of the proposed algorithm is of $\mathcal{O}(N^2)$ (the complexity analysis is not provided due to the space limitations), which is significantly lower than $\mathcal{O}(N!)$ of the exhaustive search.

V. CONCLUSIONS

In this paper, we proposed a joint bit and power loading algorithm that maximizes the OFDM SU throughput and min-

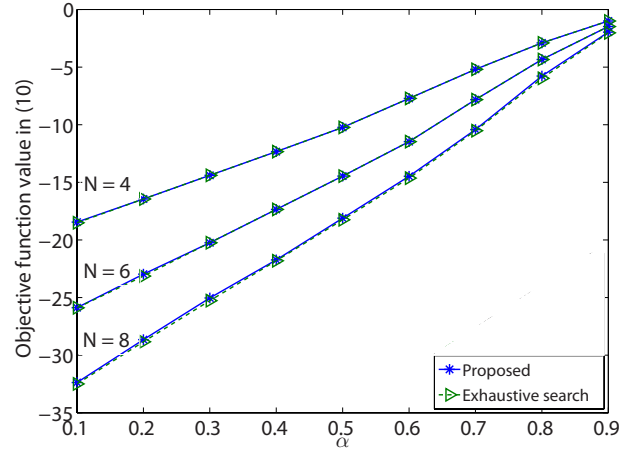


Fig. 6: Objective function for the proposed algorithm and the exhaustive search for $\mathcal{P}_{th} = 5$ μ W, $\mathcal{P}_{CCI} = \mathcal{P}_{ACI} = 10^{-10}$ μ W.

imizes its transmit power while guaranteeing a target BER and a total transmit power threshold for the SU, and ensuring that the CCI and ACI are below certain thresholds with predefined probabilities. Unlike most of the work in the literature, the proposed algorithm does not require instantaneous channel information feedback between the SU transmitter and the PUs receivers. Closed-form expressions were derived for the close-to-optimal bit and power distributions. Simulation results showed the flexibility of the proposed algorithm to tune for various power and throughput levels as needed by the CR system while meeting the constraints, with low computational complexity.

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